7.10.22

Derivation of final loglikelihood test expression

$$L(\underline{r}) = \frac{p(\underline{r}\,|\,\mathcal{H}_1)}{p(\underline{r}\,|\,\mathcal{H}_0)}$$

→ write out and substitute σ_0^2, σ_1^2 and $\hat{h}$

$\hat{h} = \frac{p^H r}{N_p}$

is real by def. of $\hat{h}$: $\|r\|^2 - 2\mathrm{Re}\{\hat{h}\, p^H r\} + |h|^2 \cdot \|p\|^2$

$$= \frac{(\pi\sigma_0^2)^{N_p}}{(\pi\sigma_1^2)^{N_p}} \exp\left(-\frac{\|\underline{r} - \hat{h}\underline{p}\|^2}{\sigma_1^2}\right) \underbrace{\exp\left(+\frac{\|r\|^2}{\sigma_0^2}\right)}_{\exp(N_p)}, \qquad \sigma_0^2 = \frac{\|r\|^2}{N_p}$$

$\hat{h}^* = \frac{r^H p}{N_p}$, $\quad |\hat{h}|^2 = |p^H r|^2 \cdot \frac{1}{N_p^2}$, $\quad \|p\|^2 = N_p$

$$= \frac{(\pi\sigma_0^2)^{N_p}}{(\pi\sigma_1^2)^{N_p}} \exp\left(-\frac{\|r\|^2 - 2\hat{h}^* p^H r + |\hat{h}|^2 \cdot \|p\|^2}{\sigma_1^2}\right) \cdot \exp(N_p)$$

$$= \frac{(\pi\sigma_0^2)^{N_p}}{(\pi\sigma_1^2)^{N_p}} \exp\left(-\frac{\|r\|^2 - 2\frac{\|r\|^2\|p\|^2}{N_p} + \frac{|p^H r|^2}{N_p}}{\sigma_1^2}\right) \exp(N_p)$$

$$= \frac{(\pi\sigma_0^2)^{N_p}}{(\pi\sigma_1^2)^{N_p}} \exp\left(-\frac{\|r\|^2 - 2\frac{\|r\|^2\|p\|^2}{N_p} + \frac{|p^H r|^2}{N_p}}{\sigma_1^2}\right) \exp(N_p)$$

$\|r\|^2 - 2\|r\|^2 + \frac{|p^H r|^2}{N_p} = -\|r\|^2 + \frac{|p^H r|^2}{N_p} = -\left(\|r\|^2 - \frac{|p^H r|^2}{N_p}\right)$

same !

$\sigma_1^2 = \frac{1}{N_p}\left(\|r\|^2 - \frac{|p^H r|^2}{N_p}\right)$

$$= \frac{(\pi\sigma_0^2)^{N_p}}{(\pi\sigma_1^2)^{N_p}} \underbrace{\exp(N_p)\exp(N_p)}_{\text{constant.}} \rightarrow \text{IGNORE}$$